



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – NOVEMBER 2014

MT 1502 - ALGEBRA AND CALCULUS - I

Date : 07/11/2014
Time : 01:00-04:00

Dept. No.

Max. : 100 Marks

PART – A

(10 x 2 = 20)

Answer ALL questions:

1. Write the Leibnitz formula for the n^{th} derivative of a product.
2. Find the polar subtangent of the curve $r = ae^{\theta \cot \alpha}$.
3. State the conditions for maxima and minima of two variables.
4. Write the steps used in Lagrange's method of undetermined multipliers.
5. Write the Cartesian formula for the radius of curvature.
6. Define evolute of a curve.
7. Find the sum of the fourth roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$.
8. Form the equation one of whose roots is $2 + i\sqrt{3}$.
9. State Descartes's rule of signs for negative roots.
10. Write the Cardon's method of solution of a cubic polynomial.

PART – B

(5 x 8 = 40)

Answer any FIVE questions:

11. If $y = (x + \sqrt{1+x^2})^m$, Prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$.
12. Show that the parabolas $r = a \sec^2 \frac{\theta}{2}$ and $r = b \operatorname{cosec}^2 \frac{\theta}{2}$ intersect at right angle.
13. Investigate the maximum and minimum values of the function $f = 4x^2 + 6xy + 9y^2 - 8x - 24y + 4$.
14. Find the radius of curvature at 't' on the curve $x = 6t^2 - 3t^4, y = 8t^3$.
15. Find the asymptotes of curve $x^3 + 3x^2y - xy^2 - 3y^3 + x^2 - 2xy + 3y^2 + 4x + 7 = 0$.
16. Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$, given that $1 + \sqrt{-1}$ is a root of it.
17. Diminish the roots of the equation $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ by 3.
18. Solve $x^4 + 20x^3 - 143x^2 + 430x + 462 = 0$ by removing the second term.

Answer Any TWO Questions:

19. a). Find the angle between the radius vector and the tangent for the curves

(i) $r = a(1 + \cos \theta), \theta = \frac{\pi}{4}$ (ii) $r^2 = a^2 \cos 2\theta, \theta = \frac{\pi}{6}$.

b) Using Lagrange's multipliers method find the maximum and minimum values of $f(x, y) = x^2 - y^2$ subject to $x^2 + y^2 = 1$. (10 + 10)

20. Obtain the evolute of the curve $x = a(\cos \theta + \theta \sin \theta), y = a(\sin \theta - \theta \cos \theta)$.

21. a). If the sum of the two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ equals the sum of the other two, prove that $p^3 + 8r = 4pq$.

b). Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$. (10 + 10)

22. a). Show that $3x^5 - 2x^3 - 4x + 2 = 0$ has three real and two imaginary roots.

b) Using Horner's method find the real root of $x^3 + 6x - 2 = 0$. (5 + 15)

\$\$\$\$\$\$